Numerical Approach Using Only Meshless Method for Solving Steady-State Scattering Problems of Electromagnetic Wave

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The hybrid approach of the domain-type and the boundary-type meshless methods is applied to the steady-state scattering problems of electromagnetic wave. In addition, the performance of the proposed method is investigated numerically. The results of computations show that the convergent rate is almost the same value regardless of the wave number, and the accuracy of the proposed method is degraded with an increase in the value of the wave number. In addition, the GMRES method without restart is a useful solver for a large-scale simulation.

***Index Terms*—Boundary value problems, integral equations, numerical analysis, partial differential equations.**

I. Introduction

RECENTLY, the steady-state scattering problems of electromagnetic wave has been solved by using the hybrid method [1], [2]. In the hybrid method, the finite element method (FEM) and the boundary element method (BEM) are applied to internal problem and the external one, respectively. Thereafter, the resulting linear systems are combined. On the other hand, a target domain, a boundary and an interface must be divided into a set of elements for obtaining the solution in conventional method. This process is time-consuming in the case where the object shape is complex.

In order to resolve the disadvantage of the element-based methods such as the FEM and the BEM, some meshless methods have been proposed [3]–[6]. The meshless methods can classify as either the domain-type method or the boundary-type one. If the domain-type meshless method or the boundary-type one are applied instead of the FEM and the BEM, the hybrid method without the mesh generation can be proposed.

The purpose of the present study is to develop the numerical approach using only meshless method for solving the steady-state scattering problems of the electromagnetic wave and to investigate the performance numerically.

II. Numerical Approach

For simplicity, we consider the steady-state scattering problem of electromagnetic waves from a cylindrical objects of an arbitrary cross section. In addition, we assume an TM wave is incident on the normal direction to the axis of the cylinder. In other words, we deal with the following problem:

$$-\left(\Delta+k^2\right)E_z=0, \qquad \text{in } \Omega_{\mathrm{I}} \tag{1}$$

$$-\left(\Delta+k^2\right)E_z=E_0\,\delta\left(\boldsymbol{x}-\boldsymbol{x}_{\mathrm{s}}\right), \qquad \text{in } \Omega_{\mathrm{E}} \tag{2}$$

where Ω_{I} and Ω_{E} denote a domain bounded by a simple closed curve $\partial\Omega$ and an infinite domain which encloses Ω_{I}, respectively. Furthermore, E_z, E_0 and k denote the z-axis component of total electric field, the amplitude of incident electric field and a wave number, respectively. Moreover, $\delta\left(\boldsymbol{x}-\boldsymbol{x}_{\mathrm{s}}\right)$ denotes the two-dimensional Dirac delta function, and $\boldsymbol{x}_{\mathrm{s}}$ is a point at which a source of electric field located. As the boundary condition, we give the following equations:

$$[\![E_z]\!]=0, \quad \text{on } \partial\Omega,$$

$$\left[\!\!\left[\frac{\partial E_z}{\partial n}\right]\!\!\right]=0, \quad \text{on } \partial\Omega$$

where $\boldsymbol{n}$ indicate an unit normal on $\partial\Omega$. Furthermore, $[\![\ \ ]\!]$ means the operator which denotes a gap of operand across the boundary. Throughout the present study, the boundary-value problem of (1) and that of (2) are called the internal problem and the external one, respectively.

Before discretizing, we must derive both a weak form in the internal problem and a boundary integral equation in the external problem. By assuming that the Dirichlet boundary condition is imposed on $\partial\Omega$, we can get the following weak form:

$$\forall w \text{ s.t. } w\big|_{\partial\Omega}=0: J[w,E_z]=0, \tag{4}$$

where $J[w,u]$ is the functional defined by

$$J[w,u]\equiv\iint_{\Omega_{\mathrm{I}}}\nabla w\cdot\nabla u\,d^2\boldsymbol{x}-k^2\iint_{\Omega_{\mathrm{I}}}w\,u\,d^2\boldsymbol{x}.$$

Moreover, $\forall w \text{ s.t. } w\big|_{\partial\Omega}=0$ denotes an arbitrary function $w\left(\boldsymbol{x}\right)$ that fulfills $w=0$ on $\partial\Omega$. By assuming that the Sommerfeld radiation condition is satisfied at infinity, (2) is transformed to be equivalent to the boundary integral equation. Its explicit form is given by

$$c(\boldsymbol{y})E_z(\boldsymbol{y})+\oint_{\partial\Omega}\frac{\partial w^*\left(\boldsymbol{x}(s),\boldsymbol{y}\right)}{\partial n_{\mathrm{E}}}E_z(\boldsymbol{x}(s))ds - \oint_{\partial\Omega}w^*\left(\boldsymbol{x}(s),\boldsymbol{y}\right)\frac{\partial E_z(\boldsymbol{x}(s))}{\partial n_{\mathrm{E}}}ds=E_0\,w^*(\boldsymbol{x}_{\mathrm{s}},\boldsymbol{y}), \tag{5}$$

where $c(\boldsymbol{y})$ is a shape coefficient and $w^*(\boldsymbol{x},\boldsymbol{y})$ is the fundamental solution of $-(\Delta+k^2)$. Furthermore, s denote an arclength along $\partial\Omega$. Throughout the present study, we apply the collocation EFGM [5] and the X-BNM [6] to the internal problem and the external one, respectively.

In order to discretize the weak form (4), the boundary integral equation (5) and the associated boundary conditions, the N nodes are placed in $\Omega_{\rm I} \cup \partial\Omega$. Subsequently, the MLS shape functions $\phi_i(\boldsymbol{x})$ $(i = 1, 2, \cdots, N)$ are assigned to all nodes. Similarly, the RPIM shape functions $\psi_p(s)$ $(p = 1, 2, \cdots, M)$ are assigned to nodes on $\partial\Omega$. Here, M denotes the number of nodes on $\partial\Omega$.

Under the above assumptions, (4), (5) and its associated boundary conditions are discretized to the linear system. By solving the linear system, we can obtain the distributions of the solution on $\partial\Omega \cup \Omega_{\rm I}$. Note that the resulting linear system has not a diagonal-dominant coefficient matrix. Furthermore, its matrix also becomes complex and asymmetric. Hence, we cannot solve it by using stationary iterative methods. For this reason, the GMRES method for the complex linear system has been adopted as the solver.

III. Performance Evaluation

In this section, we investigate the performance of the proposed method. Throughout the present section, the boundary $\partial\Omega$ is assumed as $\partial\Omega = \left\{(x, y) | x^2 + y^2 = 4\right\}$. Furthermore, the analytic solution of target problem is given by

$$\frac{E_z}{E_0} = -\frac{i}{4} H_0^{(2)} \left(k\sqrt{x^2 + (y-3)^2} \right),$$

where $H_0^{(2)}(x)$ denotes the Hankel function of the second kind for integer order zero. In this study, E_0 is fixed as $E_0 = 1$

The nodes are uniformly placed in $\Omega_{\rm I} \cup \partial\Omega$. Moreover, weight function $w_i(\boldsymbol{x})$ used in the MLS shape function is given by

$$w_i(\boldsymbol{x}) = w\left(|\boldsymbol{x} - \boldsymbol{x}_i|\right),$$

$$\rho(\bar{r}) = \begin{cases} \frac{\exp[-(\bar{r}/c_1)^2] - \exp[-\left(\bar{R}/c_1\right)^2]}{1 - \exp[-\left(\bar{R}/c_1\right)^2]} & (\bar{r} \le \bar{R}) \\ 0 & (\bar{r} > \bar{R}). \end{cases}$$

Here, $\bar{R}$ and c_1 indicate a support radius and a constant, respectively. In this study, c_1 is assumed to be equal to the minimum distance h between two nodes and $\bar{R}$ is fixed as $\bar{R} = 2.5h$. In addition, the radial basis function $r_i(s)$ used in the RPIM shape function is given by

$$r_p(s) = \rho\left(|s - s_p|\right),$$

$$\rho(\bar{r}) = \begin{cases} \exp[-c_2\, \bar{r}^2] & (\bar{r} \le R_p) \\ 0 & (\bar{r} > R_p). \end{cases}$$

where s_p denotes the arclength to the pth boundary node. Furthermore, R_p is defined by

$$R_p = \gamma \min\left(\left| s_{\bmod (p+1,\, M)} - s_p \right|, \left| s_{\bmod (p-1,\, M)} - s_p \right| \right).$$

Here, c_2 and γ is constants and these parameters are fixed as $c_2 = 1, \gamma = 1.15$.

Let us investigate the accuracy of the proposed method. The relative errors are calculated as a function of N and are depicted in Fig. 1. We see from this figure that the relative errors are almost proportional to $N^{-0.53}$. This tendency does not depend on the value of k. In addition, the accuracy of the proposed method becomes low with an increase in the value of k.

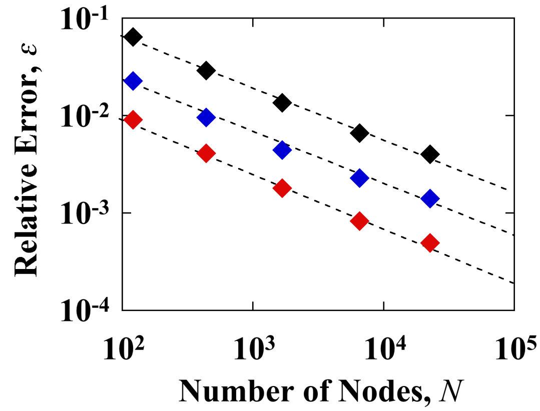

Fig. 1. Dependence of the relative error ε on the total number N of nodes. Here, the black, blue and red symbols denote $k = 1$, $k = 0.5$ and $k = 0.1$, respectively.

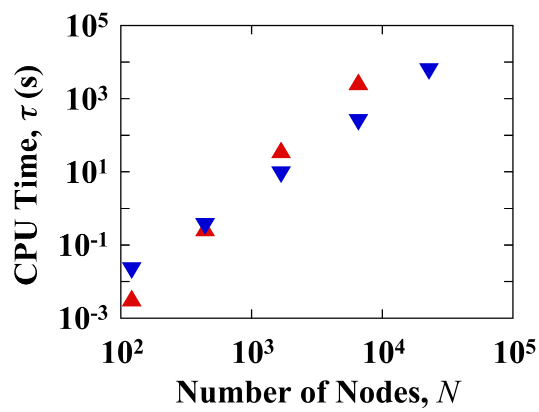

Fig. 2. Dependence of the CPU time τ (s) required for the solver on the number N of nodes. Here, the blue and red symbols denote the GMRES method without restart and *LU* factorization, respectively.

Next, we evaluate the solver speed of the proposed method by comparing with *LU* factorization. The CPU times are plotted as a function of N in Fig. 2. This figure indicates that the GMRES method is useful solver for a large-scale simulation.

As the future work, we will apply the proposed method to the problem with the complex boundary shape.

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